On Certain Aspects of Strain Rate Sensitivity of Sheet Metals

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The formability of a material depends upon the strain hardening and strain rate hardening of the material. In this study, constitutive parameters using the power law constitutive equation are determined for six different strength steels and two aluminum alloys over different strain ranges, including approximations of the postuniform elongation range. Constitutive parameters are found to be different at different strain ranges. The strain hardening of steels increases with strain at low strain levels (less than 5%) and decreases at high strain levels (greater than 10%). Strain rate hardening decreases with strain for all steels and aluminum alloys. Uniform elongation depends only on strain hardening, and postuniform elongation depends only on strain rate hardening. However, the total elongation depends on both strain hardening and strain rate hardening.

Keywords

strain hardening, strain rate hardening, tensile test, mechanical properties, sheet steel, sheet aluminum, constitutive equation

1. Introduction

STRAIN hardening and strain rate hardening are two primary sources of hardening during deformation for sheet metals. The formability of a material is primarily governed by the coupling effect of these two hardening parameters. Strain hardening and strain rate hardening enhance the ability of a material to resist localized necking and to distribute strain uniformly during forming.

The necking resistance of a material with strain hardening and strain rate hardening results in a higher forming limit. Theoretical modeling (Ref 1-3) showed that the plane strain intercept of the forming limit curve (FLC) (Ref 4, 5) increased with an increase of strain hardening and strain rate hardening of the material. Hecker (Ref 6) showed that the FLC for a mild steel was much higher than for an aluminum alloy although they had similar strain hardening. Unlike a near-zero strain rate sensitivity of the aluminum alloy, the positive strain rate sensitivity of the steel delayed localized necking and thus resulted in a higher forming limit than for aluminum.

Another contribution of strain hardening and strain rate hardening to formability of a material is the ability to distribute strain uniformly. In press forming, gradients in strain and strain rate are always developed due to frictional and geometric constraints. Such nonuniformity of thinning causes different strains and strain rates in different areas during deformation. A higher strain rate in a particular area results in higher strain in that area for a given press speed. Due to the strain hardening and strain rate hardening of the material, further deformation in this area would require higher stress or force in the sheet plane. This higher stress or force requirement would shift the deformation to adjacent areas, which require less stress or force to further deform. This would lead to more uniform strain distribution overall. However, the effect of strain rate hardening on the uniformity of strain distribution is significantly greater than that of strain hardening because the strain rate acts through the time rate of strain rather than through strain itself (Ref 7). For example, the strain distribution of the above-mentioned mild steel and the aluminum alloy in a deep drawing process showed that the strain gradient of aluminum was greater than that of steel at the same draw depth (Ref 6).

Higher yield strength provides better dent resistance (Ref 8-12). For strain hardening and strain rate hardening materials, the yield strength of the material increases with the increase in strain and strain rate (Ref 11-13), which would provide better dent resistance.

In this study, strain hardening and strain rate hardening properties of different strength steels and aluminum alloys are provided over different strain ranges. Strain rate sensitivity for several different strength steels is also measured in both the as-received and prestrained strain states. The variation of strain rate sensitivity with strain in the tensile test is studied. Dependence of elongations (uniform, postuniform, and total) on strain hardening and strain rate hardening is also determined. Effects of strain hardening and strain rate hardening on mechanical properties of the material are examined. Some properties of necking resistance and dent resistance for positive strain rate sensitive material are confirmed in steels.

2. Experimental Design

2.1 *Materials and Prestrain States*

Eight different materials were used in this investigation: an interstitial-free (IF) steel, an aluminum killed drawing quality (AKDQ) steel, two body-in-white high strength steels (HSS), a high strength low alloy (HSLA) steel, a bake hardenable (BH)

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Table 1 Mechanical properties of materials tested

Material Identification	Material type	Yield strength MPa	Tensile strengthMPa	Total elongation %	n-value	$\bar{r}$	Thickness mm
A	IF	152	308	44.3	0.228	1.91	0.69
B	AKDQ	181	325	42.2	0.226	1.41	0.84
C	HSS I	265	404	33.8	0.180	1.55	0.77
D	HSS II	249	392	37.3	0.210	1.43	0.81
E	HSLA	389	509	23.9	0.144	1.20	0.89
F	BH	220	354	36.6	0.175	1.45	0.86
G	2036 T4	209	346	22.7	0.224	0.60	1.05

Table 2(a) Chemical compositions for aluminum alloys

	Composition, wt%				
Material	Si	Fe	Cu	Mn	Mg
2036 T4	0.25	0.25	2.50	0.25	0.45
6111 T4	0.85	0.20	0.75	0.20	0.72

Table 2(b) Chemical compositions for steels

	Composition, wt%												+
Material	C	Mn	P	S	Si	Cu	Ni	Cr	Mo	Sn	Al	N	Cb	Ti
IF	0.009	0.22	0.013	0.005	0.01	0.01	0.02	0.019	0.007	0.007	0.036	0.0063	...	...
AKDQ	0.04	0.27	0.015	0.016	0.02	0.03	0.01	0.037	0.007	0.008	0.063	0.0057	...	...
HSS I	0.07	0.59	0.059	0.010	0.02	0.03	0.02	0.031	0.008	0.008	0.057	0.0248	...	...
HSS II	0.06	0.47	0.064	0.009	0.01	0.04	0.02	0.034	0.009	0.009	0.058	0.0145	...	...
HSLA	0.06	0.53	0.013	0.008	0.05	0.05	0.02	0.056	0.011	0.010	0.062	0.0064	0.042	0.038
BH	0.047	0.22	0.011	0.016	0.02	0.03	0.01	0.031	0.006	0.008	0.081	0.0085	...	...

steel, and two aluminum alloys (2036 T4 and 6111 T4). The typical mechanical properties measured in an ASTM standard tensile test (Ref 14) and the chemical compositions are listed in Tables 1 and 2, respectively.

The strain rate tests were carried out for all steels at various prestrain states (as-received, 2% and 5% plane strain, and balanced biaxial stretch) and for aluminum alloys at as-received strain state. The prestrains were obtained by the Marciniak cup test (Ref 12). A subsize tensile specimen (Ref 14) cut from the center of the Marciniak cup bottom was then used in the strain rate tests.

The concept of equivalent prestrain is introduced to represent the strain levels in different strain states in order to study the effects of various prestrains on the strain and strain rate hardening of the material. This allows direct comparison of the "total strain" represented in different strain states by different amounts of strain. By using Hill's 1979 anisotropic yield criterion (Ref 15) with a stress exponent of 2, the equivalent prestrain can be expressed as:

$$\bar{\varepsilon} = \sqrt{\frac{1}{2}(1+r)\left[\frac{1}{1+2r}(\varepsilon_1-\varepsilon_2)^2+(\varepsilon_1-\varepsilon_2)^2\right]} \quad \text{(Eq 1)}$$

where $\bar{\varepsilon}$ is the equivalent strain, ε_1 and ε_2 are two principal strains in the sheet plane, and r is the average anisotropy value. Specifically, for a balanced biaxial stretch strain state, the equivalent strain is then obtained as:

$$\bar{\varepsilon} = \sqrt{2(1+\bar{r})}\varepsilon_B \quad \text{(Eq 1a)}$$

where ε_B is the true strain (major or minor) in the balanced biaxial strain state. For a plane strain state,

$$\bar{\varepsilon} = \sqrt{\frac{1}{1+2\bar{r}}}(1+\bar{r})\,\varepsilon_{PS} \quad \text{(Eq 1b)}$$

where $\varepsilon_{P.S.}$ is the true major strain at plane strain.

2.2 *Test Method*

A continuous constant strain rate test described in a recent article (Ref 16) was used in this study. In order to study the effect of different strain rate hardening on the mechanical properties, six different strain rates were used at each prestrain state for a given material. These strain rates were obtained using different crosshead speeds in the tensile machine. Each individual specimen was pulled at one constant crosshead speed throughout the test. The crosshead speeds of 0.21 mm/s, 1.33 mm/s, 13.32 mm/s, 66.67 mm/s, 128.67 mm/s, and 257.33 mm/s were used in the six tensile tests for a given material and prestrain

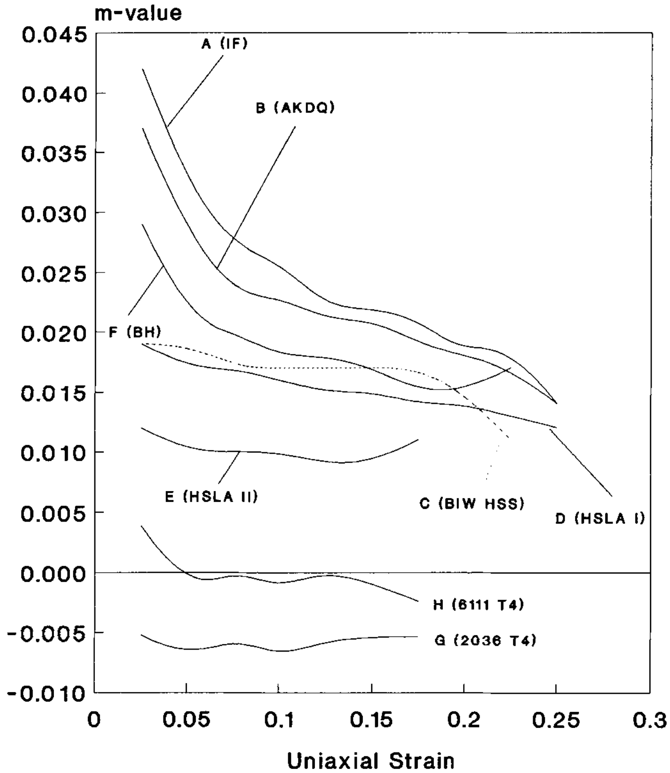


Fig. 1 Variation of m-value with uniaxial strain

state. The corresponding strain rates are approximately 0.002/s, 0.01/s, 0.1/s, 0.5/s, 1.0/s, and 2.0/s, respectively, for these crosshead speeds.

3. Strain Rate Sensitivity of Sheet Metals

3.1 *Definition of Strain Rate Sensitivity*

Conventionally, strain rate sensitivity is expressed by:

$$\sigma = K_0\dot{\varepsilon}^m \quad \text{(Eq 2)}$$

where σ is the true stress; K_0 is, in general, a function of strain (ε) and temperature (T); $\dot{\varepsilon}$ is the strain rate; and m is the strain rate sensitivity. For power law strain hardening materials, Eq 1 can be rewritten as:

$$\sigma = K\varepsilon^n\dot{\varepsilon}^m \quad \text{(Eq 3)}$$

where K is a constant representing the material strength, and n is the strain hardening value. Thus, the m-value can be calculated by:

$$m = \frac{\ln\sigma_2 - \ln\sigma_1}{\ln\dot{\varepsilon}_2 - \ln\dot{\varepsilon}_1} = \frac{\Delta(\ln\sigma)}{\Delta(\ln\dot{\varepsilon})}\Big|_{\varepsilon,T=\text{constant}} \quad \text{(Eq 4)}$$

where Δ represents the difference in two different strain rate tests. By taking the logarithm of both sides of Eq 2:

$$\ln\sigma = \ln K + n\ln\varepsilon + m\ln\dot{\varepsilon} \quad \text{(Eq 5)}$$

the m-value can also be determined from the slope of the regression line from the known $\ln\sigma$ and $\ln\dot{\varepsilon}$ data at a given strain. For a known set of data ($\ln\sigma$, $\ln\varepsilon$, and $\ln\dot{\varepsilon}$) measured from the constant strain rate tensile test, the n-value, m-value, and constant K can be simultaneously determined by a multiple regression statistical approach.

Although there are several other forms, such as the overstress description (Ref 7) and the strain rate dependent form (Ref 17, 18), used to describe the strain rate sensitivity of a material, the description in Eq 3 is the one used most often in sheet metal forming applications because of its simple form and its historical acceptance. Therefore, Eq 3 is used in the following analysis.

4. Experimental Results

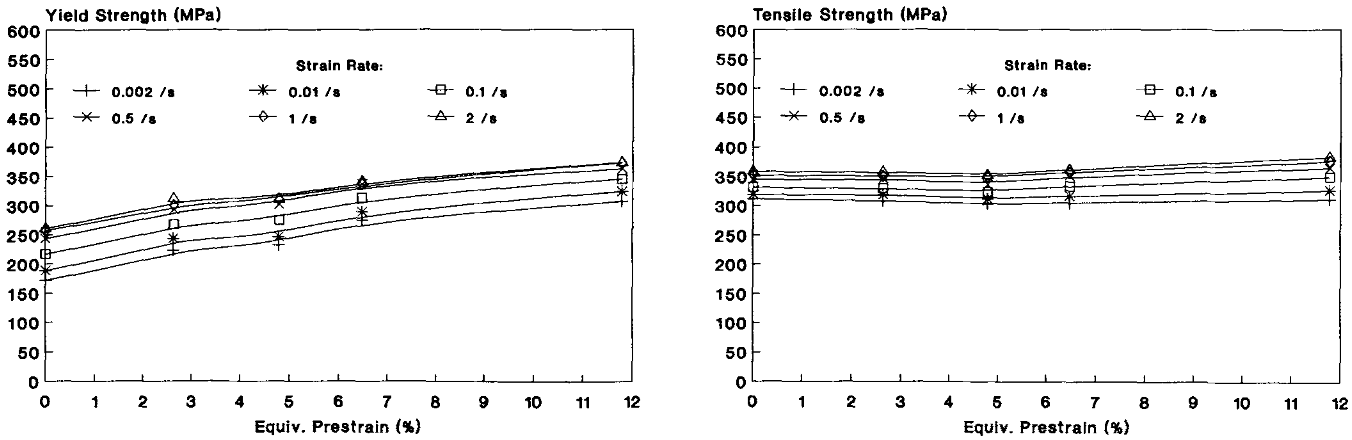
4.1 *Strain Rate Sensitivity at Different Strains*

Strain rate sensitivity at a given strain can be calculated by a least squares approach based on Eq 4 using data at different strain rates. Figure 1 shows these strain rate sensitivity values at different uniaxial strains in the as-received strain state for steels and aluminum alloys tested. Like the n-value, the m-value is different for different strength steels, and lower strength steels have higher m-value. The m-value varies with uniaxial strain for all steels and aluminum alloys and in general decreases with an increase of strain. For low strength steels like IF and AKDQ steels, strain rate sensitivity is very high at low strain levels and decreases rapidly with an increase of strain at low strain levels. Strain rate sensitivity then decreases gradually with strain after about 8 to 10% strain. For high strength steels, such as HSS and HSLA steels, however, the strain rate sensitivity decreases only slightly with an increase of strain. But, the m-value for aluminum alloys is negative except for 6111 T4 at strain less than 5%. Like high strength steels, the variation of m-value with strain for aluminum alloys is relatively small except for 6111 T4 at very low and very high strain levels.

4.2 *Constitutive Parameters over Different Strain Ranges*

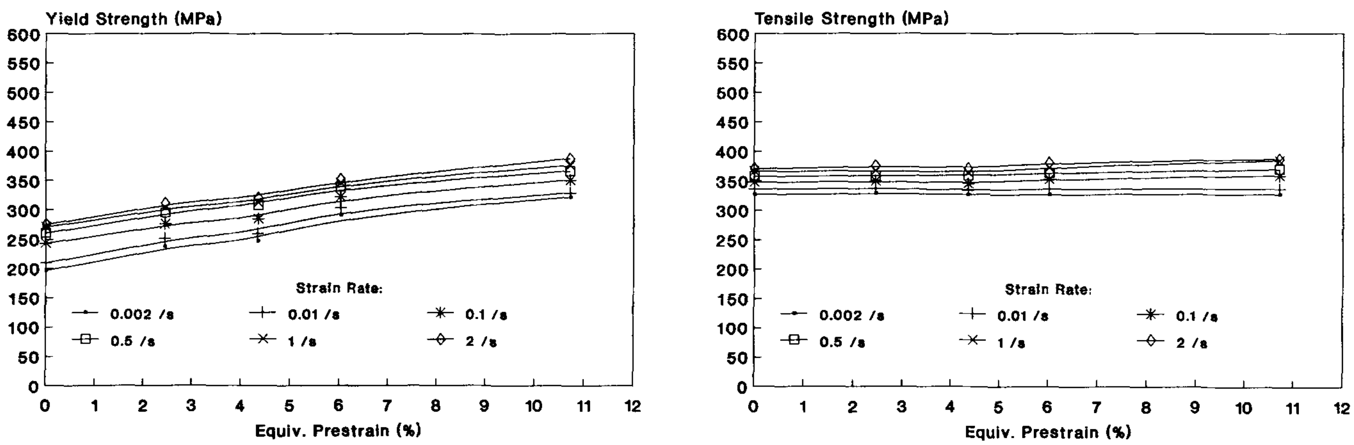
As shown in Fig. 1, strain rate sensitivity is different at different strain levels. Constitutive parameters determined over the entire uniform elongation range may misrepresent the actual material properties. In this section, constitutive parameters are determined over several different strain ranges. The appropriate parameters should be used for a specific application according to the strain range.

Constitutive parameters (n, m, K) in Eq 3 were determined using a statistical analysis approach. All data generated in tensile strain rate tests at a given strain range were used to determine these constitutive parameters. Constitutive parameters determined at strain ranges from 0.5 to 2%, from 2 to 5%, from



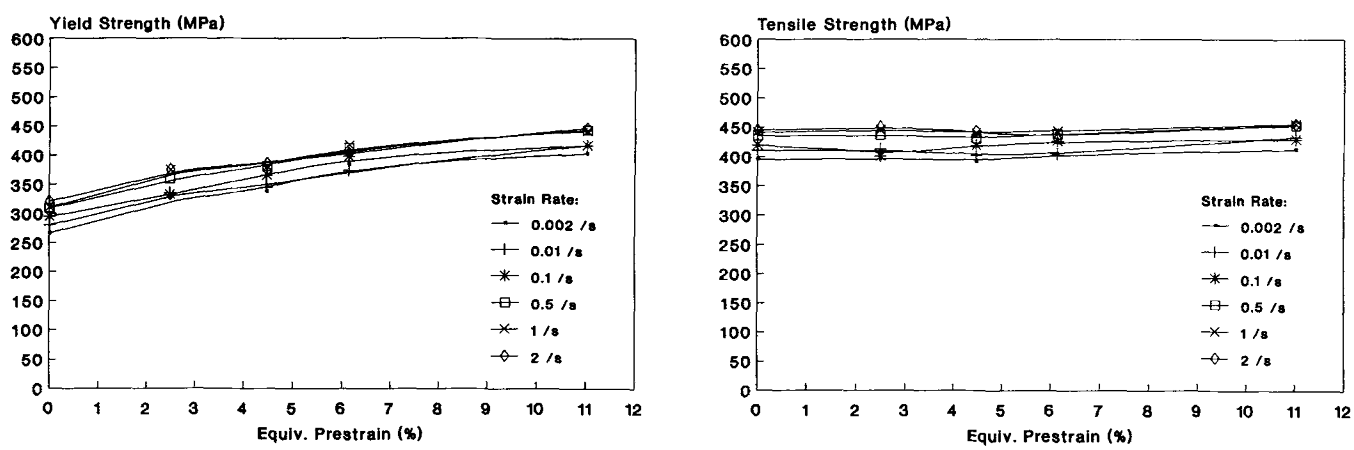
(b)

Fig. 2 Yield and tensile strength of an IF steel (A) at various prestrains and strain rates



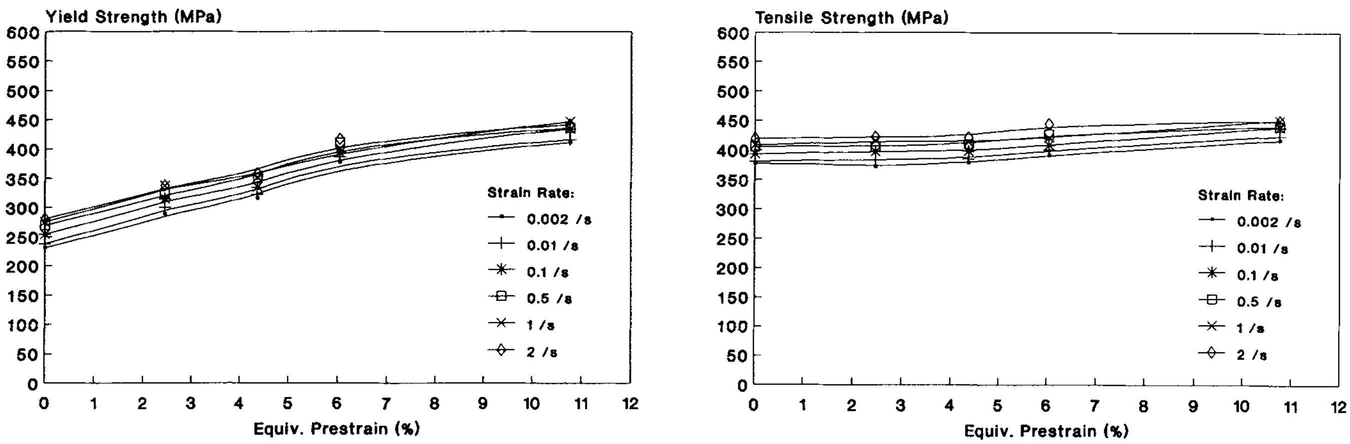
(a) (b)

Fig. 3 Yield and tensile strength of an AKDQ steel (B) at various prestrains and strain rates



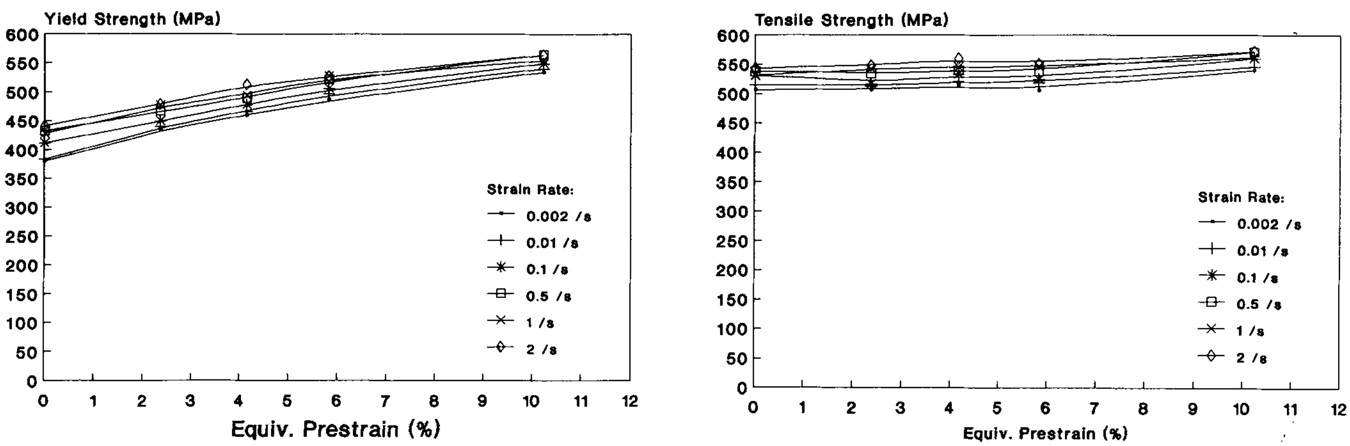
(a) (b)

Fig. 4 Yield and tensile strength of an HSS steel (C) at various prestrains and strain rates



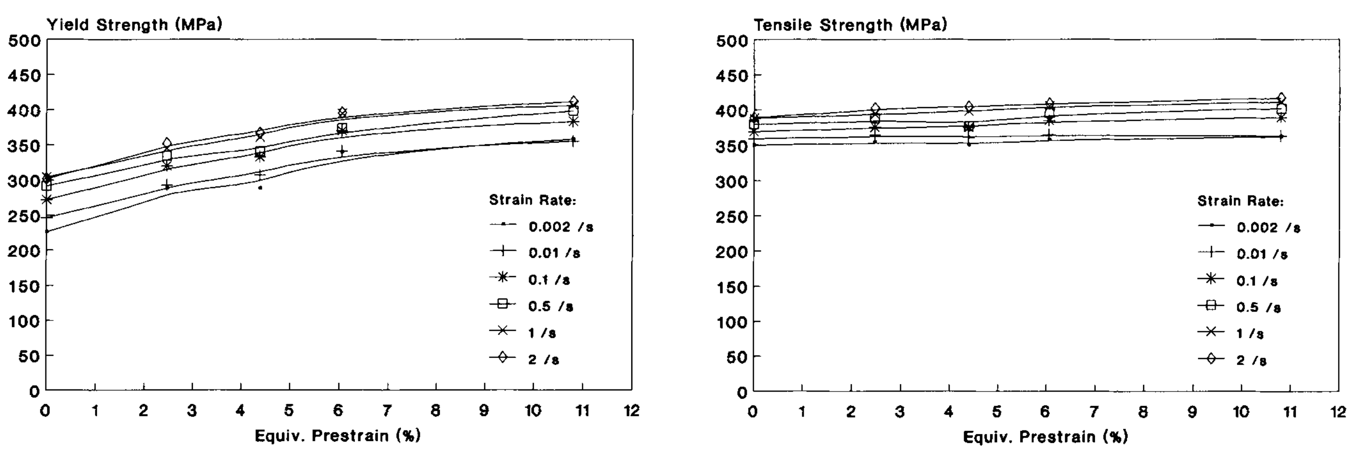
(a) (b)

Fig. 5 Yield and tensile strength of an HSS steel (D) at various prestrains and strain rates



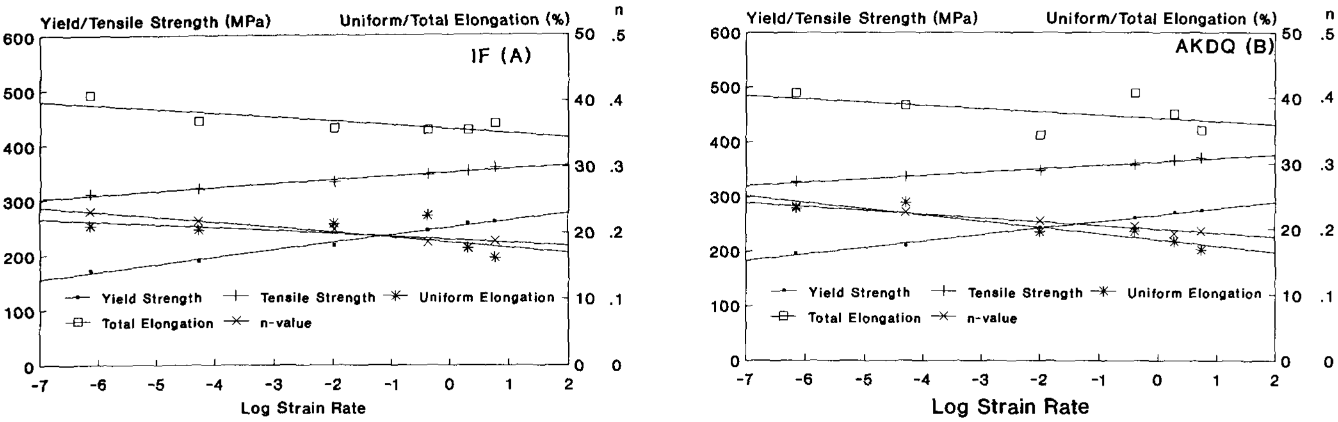
(a) (b)

Fig. 6 Yield and tensile strength of an HSLA steel (E) at various prestrains and strain rates



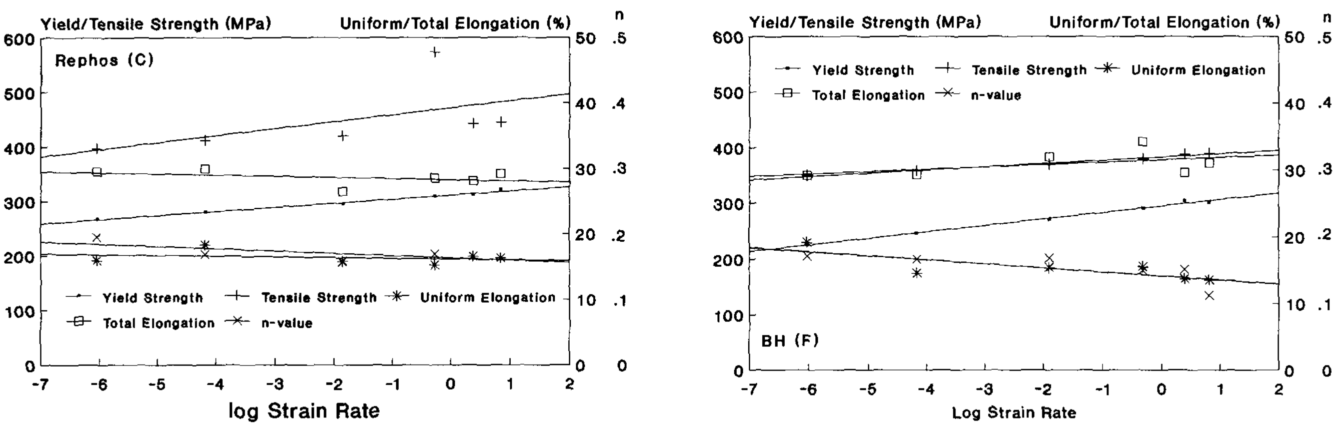
(a) (b)

Fig. 7 Yield and tensile strength of a BH steel (F) at various prestrains and strain rates



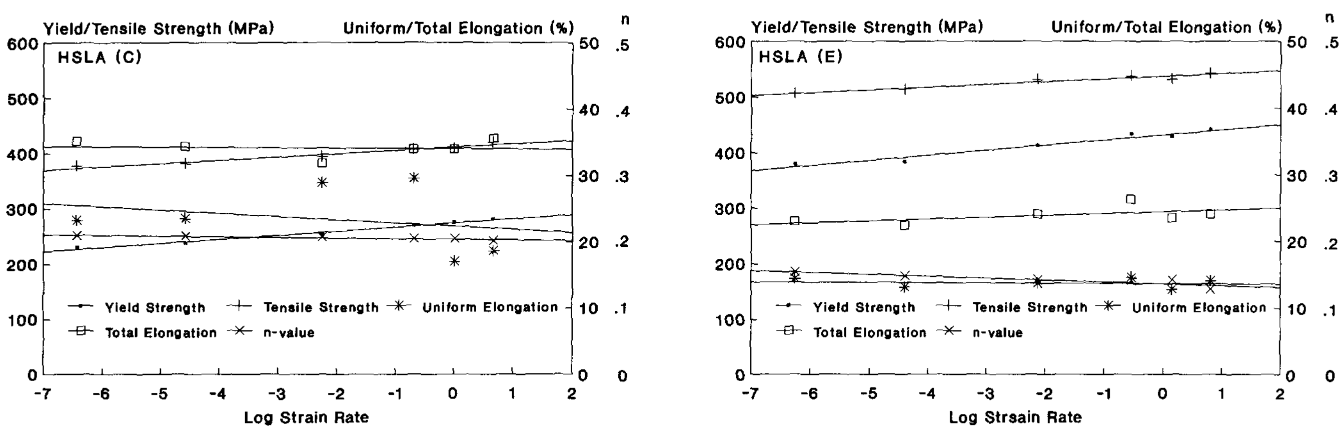
(a) (b)

Fig. 8 Effect of strain rate on mechanical properties for IF (A) and AKDQ (B) steels



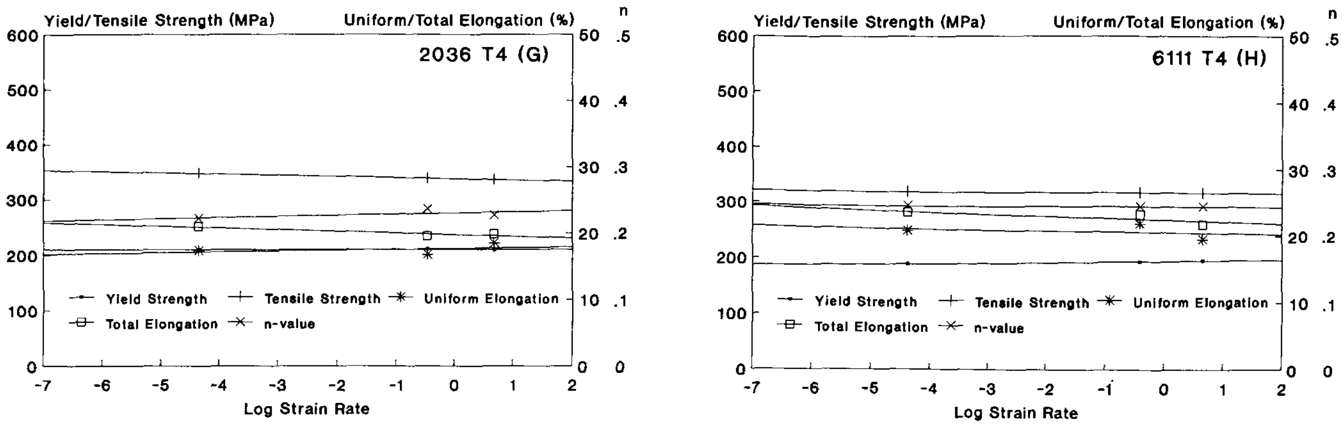
(a) (b)

Fig. 9 Effect of strain rate on mechanical properties for HSS I (C) and HSS II (D) steels



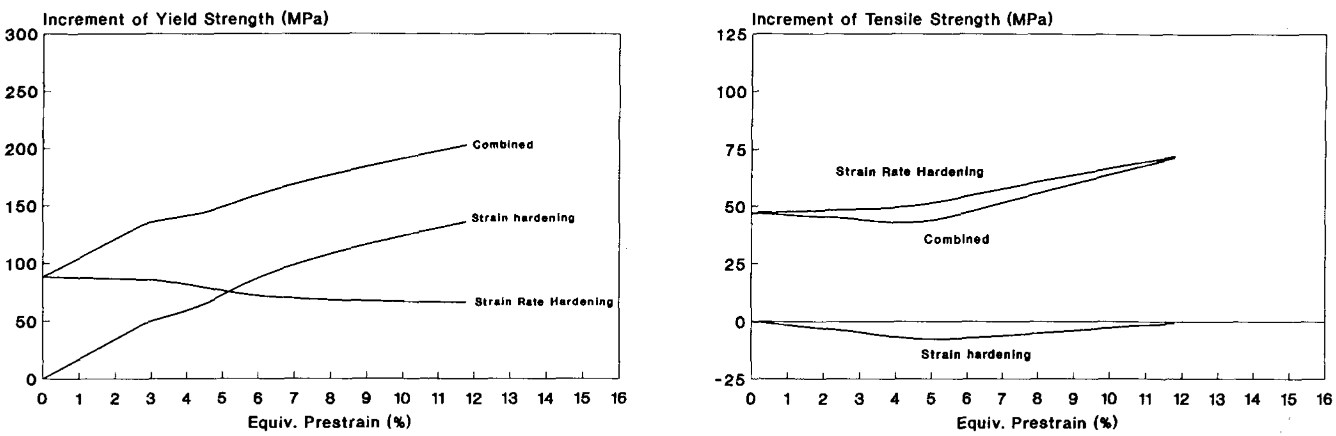
(a) (b)

Fig. 10 Effect of strain rate on mechanical properties for HSLA (E) and BH (F) steels



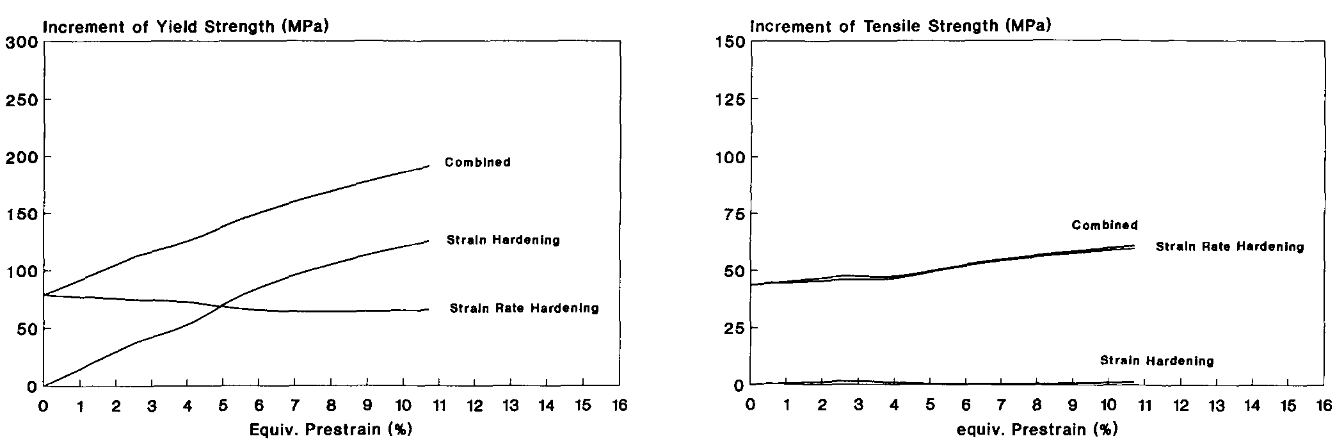
(a) (b)

Fig. 11 Effect of strain rate on mechanical properties for 2036 T4 (G) and 6111 T4 (H) aluminum



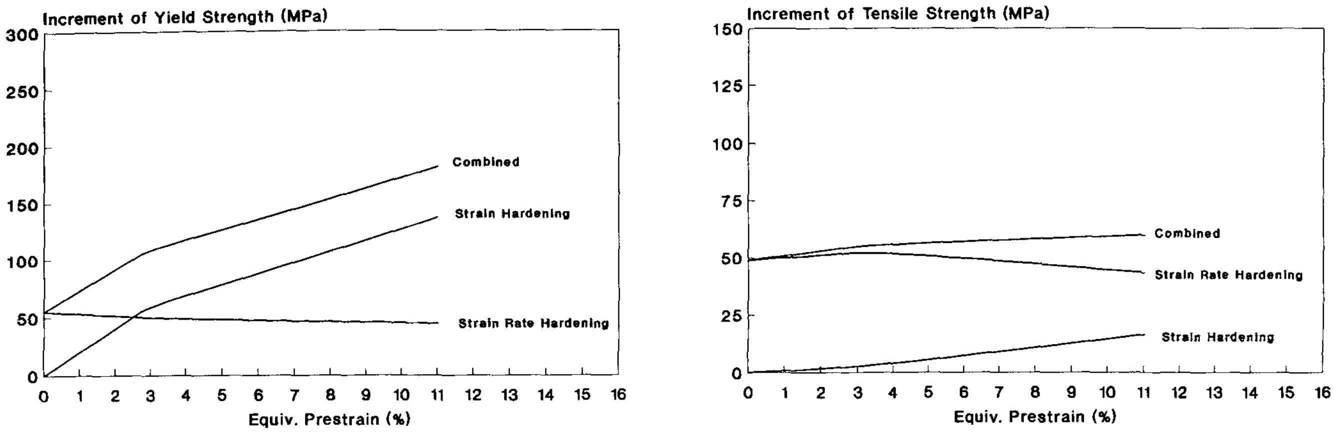
(a) (b)

Fig. 12 Effects of strain and strain rate hardening on yield and tensile strength for an IF steel (A)



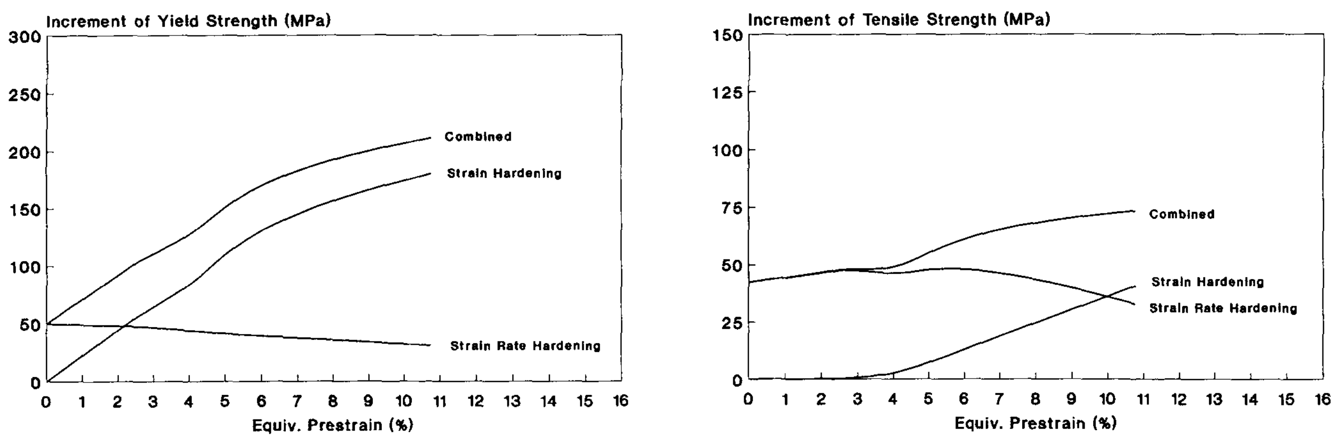
(a) (b)

Fig. 13 Effects of strain and strain rate hardening on yield and tensile strength for an AKDQ steel (B)



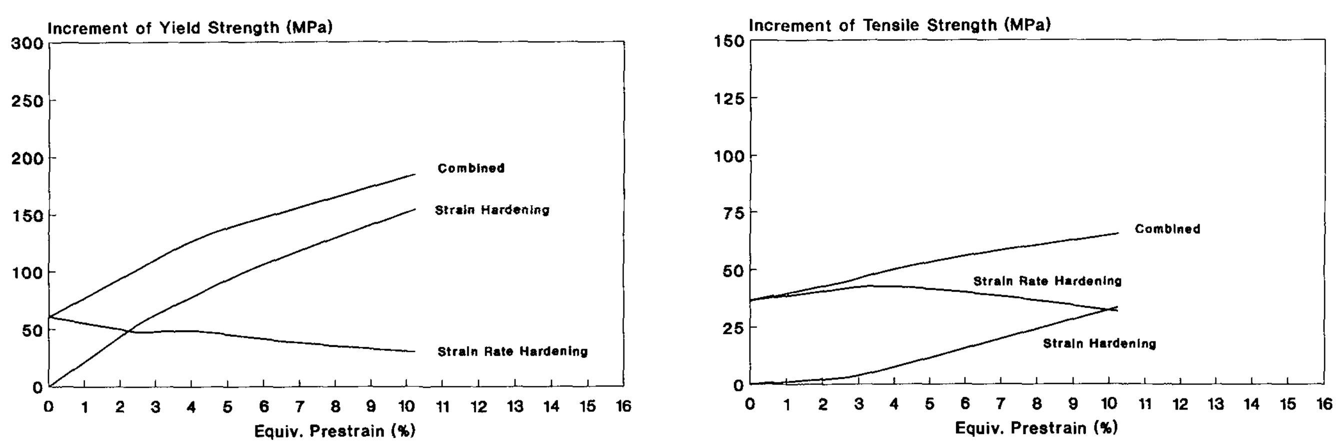
(a) **(b)**

Fig. 14 Effects of strain and strain rate hardening on yield and tensile strength for an HSS steel (C)



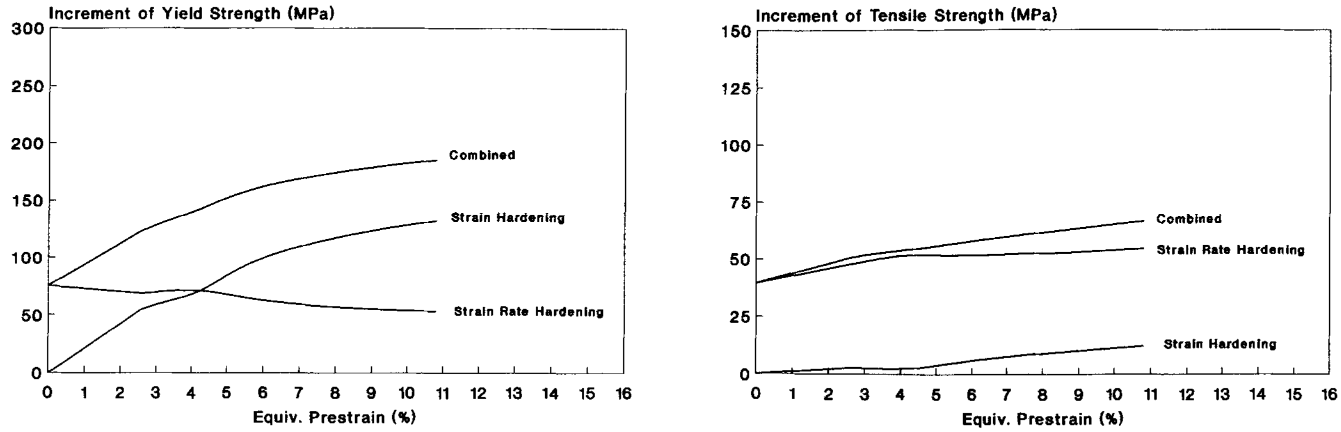
(a) **(b)**

Fig. 15 Effects of strain and strain rate hardening on yield and tensile strength for an HSS steel (D)



(a) **(b)**

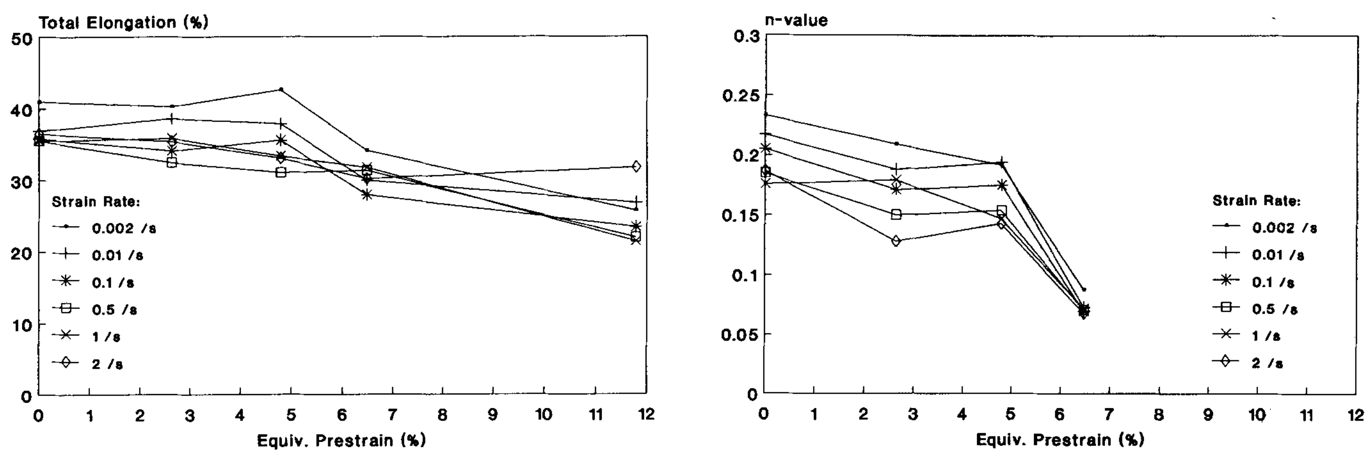
Fig. 16 Effects of strain and strain rate hardening on yield and tensile strength for an HSLA steel (E)



(a)

(b)

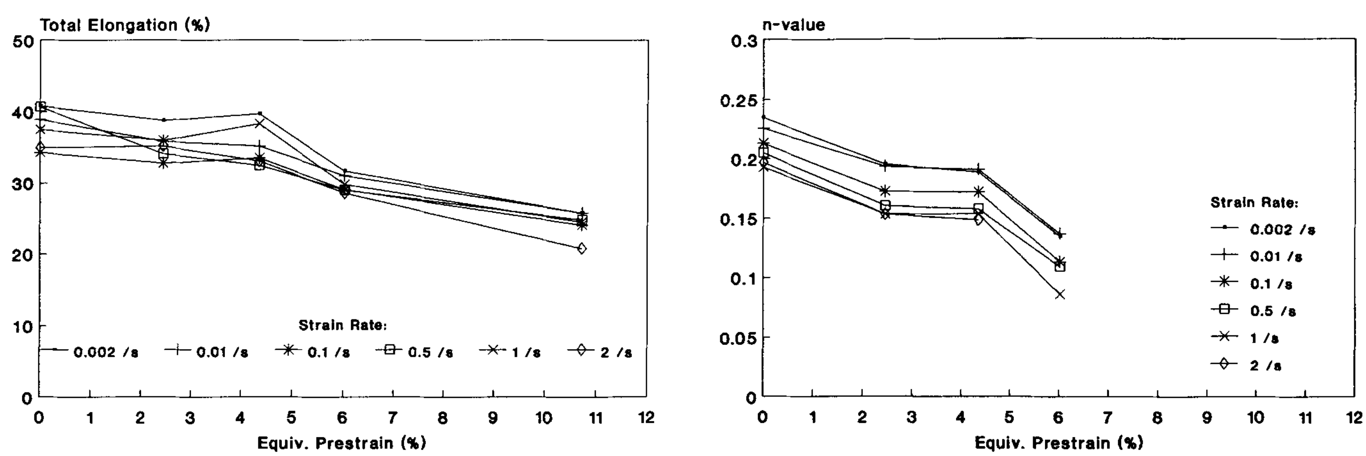
Fig. 17 Effects of strain and strain rate hardening on yield and tensile strength for a BH steel (F)



(a)

(b)

Fig. 18 Total elongation and n-value of an IF steel (A) at various prestrains and strain rates



(a)

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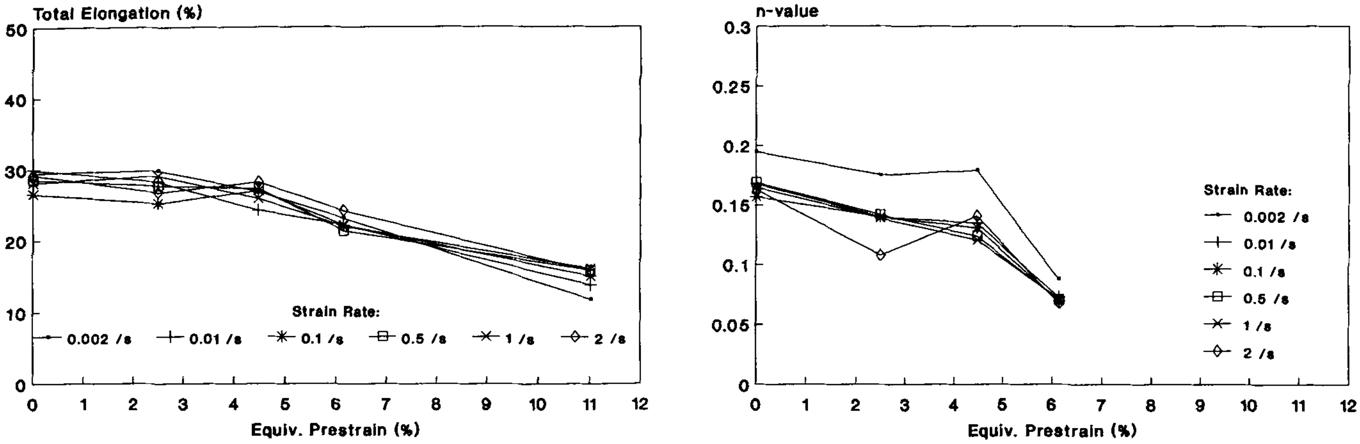
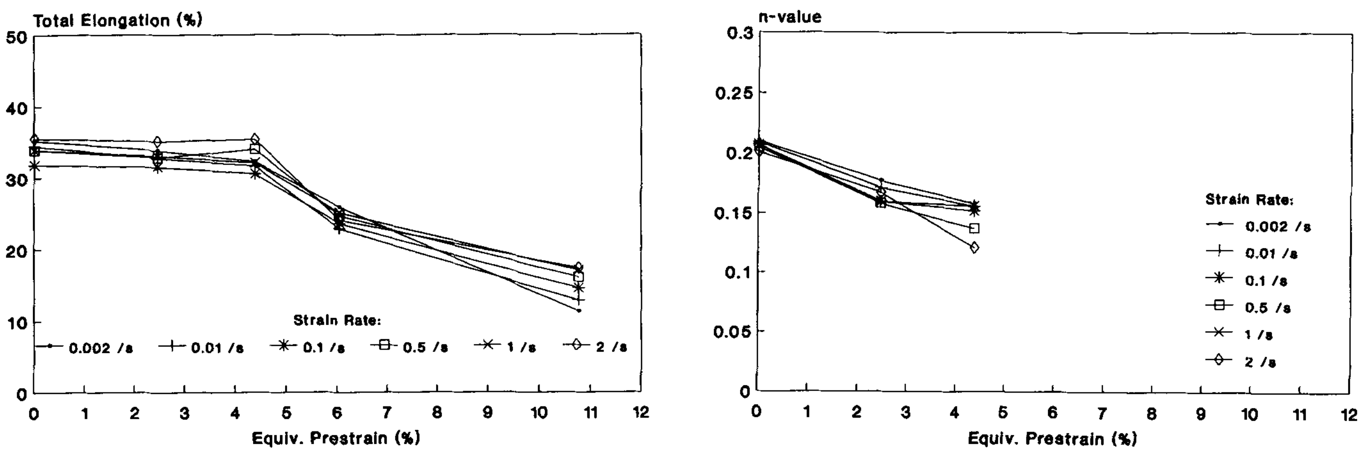
Fig. 19 Total elongation and n-value of an AKDQ steel (B) at various prestrains and strain rates



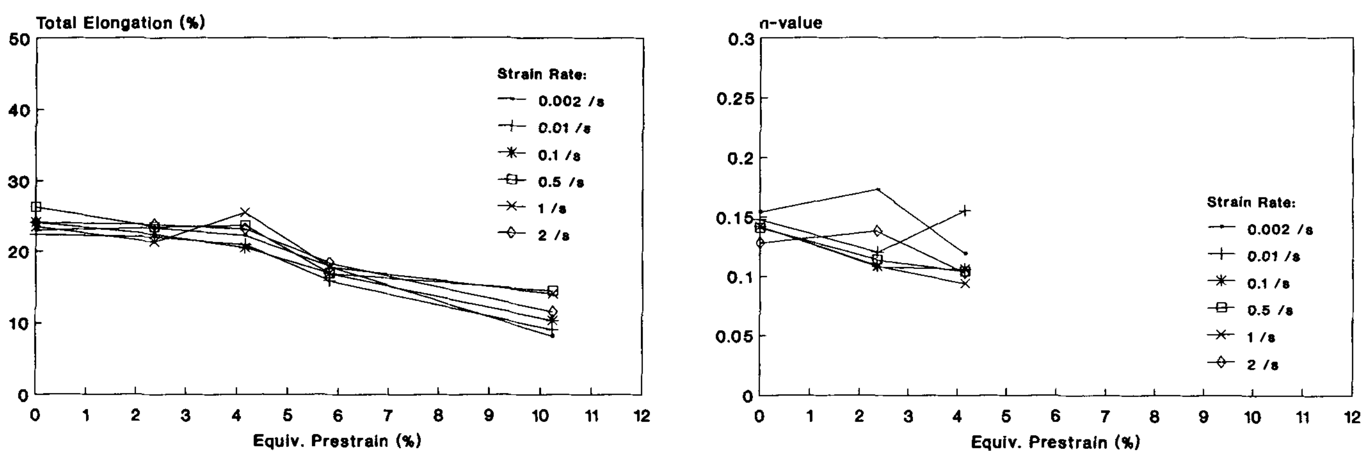
(a) (b)

Fig. 20 Total elongation and n-value of an HSS steel (C) at various prestrains and strain rates



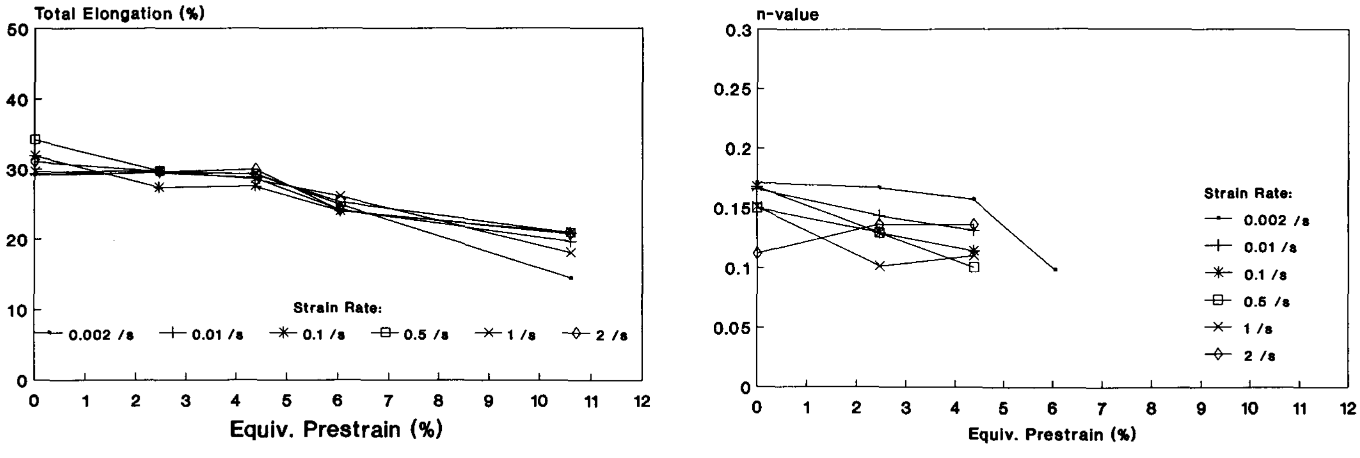
(a) (b)

Fig. 21 Total elongation and n-value of an HSS steel (D) at various prestrains and strain rates



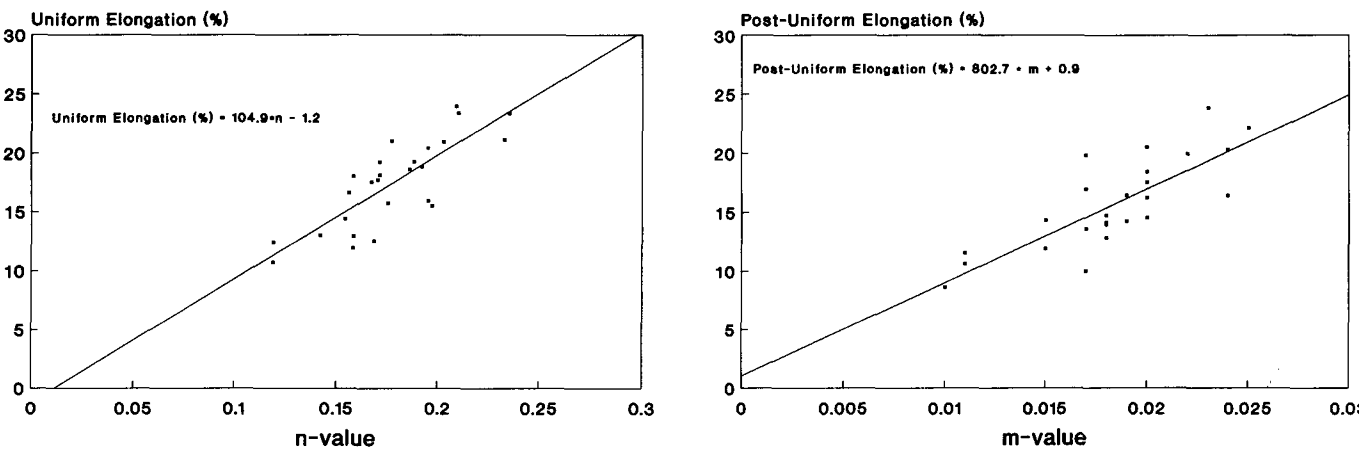
(a) (b)

Fig. 22 Total elongation and n-value of an HSLA steel (E) at various prestrains and strain rates



(a) (b)

Fig. 23 Total elongation and n-value of a BH steel (F) at various prestrains and strain rates



(a) (b)

Fig. 24 Effects of strain and strain rate hardening on uniform and postuniform elongations

5 to 10%, from 10% to uniform elongation, and from uniform elongation to one-third of postuniform elongation (postuniform elongation equals total elongation minus uniform elongation) are summarized in Table 3. Note that the values obtained after uniform elongation have not been corrected for necking, and aluminum alloys are not included because they have little postuniform elongation.

As evident, very different constitutive parameters are obtained at different strain ranges. The m-value decreases with an increase of strain for all steels and aluminum alloys tested. However, the n-value increases with an increase of strain at lower strain levels and reaches a maximum value in the strain range between 5 and 10%. Then, it decreases with a further increase of strain. These data demonstrate that a single set of constitutive parameters determined at a certain strain range are not applicable to the entire deformation history (from 0% strain to failure strain).

Also, both n-value and m-value decrease in the postuniform elongation region for steels when compared to the uniform elongation region. (Even lower n-values and m-values may be obtained if the gage length effect is considered (Ref 19, 20) because of the nonuniform deformation within the gage in the postuniform elongation region.) This implies that strain and strain rate hardening decrease in the postuniform elongation region. Therefore, in the study of localization and postlocalization, the use of constitutive equations obtained from the data in the uniform elongation region for the entire deformation history would lead to a higher limit strain. This may be, in part, why the Hill bifurcation theory (Ref 21) predicts no localized necking under biaxial stretching and why the Stören and Rice theory (Ref 22) requires a vertex type yield condition.

4.3 *Effect on Mechanical Properties*

4.3.1 *Effect on Yield Strength and Tensile Strength*

Figures 2 to 7 show the yield strength and tensile strength at different equivalent prestrains and strain rates for steels. Yield strength at various strain rates increases with prestrain. However, tensile strength also increases slightly with prestrain. This may be due to the biaxial stretch used for the prestrained specimen. Also, the yield strength and tensile strength are higher at higher strain rates.

Figures 8 to 11 explicitly show the variation of yield strength and tensile strength with strain rates at the as-received strain state for different strength steels and two aluminum alloys. Yield strength and tensile strength increase with strain rate for steels and remain almost unchanged for aluminum alloys. The increase in yield strength with increased strain indicates that the dent resistance of a panel would be significantly improved if a certain amount of strain is developed in the panel. The increase of yield strength with strain rate for steels would lead to improved dent resistance for steels in a dynamic (high rate) denting situation.

As shown above, yield strength and tensile strength of all steels increase with both strain hardening and strain rate hardening. To understand how strain hardening and strain rate hardening contribute to the final yield strength and tensile strength of the material, Fig. 12 through 17 show the increment of yield strength and tensile strength from strain hardening and strain rate hardening. In these figures, strain hardening is calculated by the value at a prestrain state minus the value at the as-received strain state at a strain rate of approximately 0.002/s. Strain rate hardening is obtained by subtracting the value at the strain rate of approximately 0.002/s from that at the strain rate of approximately 2.0/s for a given prestrain state.

The amount of contribution from strain rate hardening to the yield strength is initially greater than that from strain hardening. At a certain equivalent prestrain level, the contribution from both strain hardening and strain rate hardening is the same. When the prestrain level increases further, the increment from strain hardening exceeds that from strain rate hardening. The increment of yield strength from strain rate hardening decreases slightly with equivalent prestrain, and that from strain hardening increases steadily from prestrain.

The increment of tensile strength is dominated by strain rate hardening, particularly at low strain levels. The increment from strain hardening is almost zero for AKDQ steel and is even slightly less than zero for IF steel. The higher tensile strength at higher strain rates for steels would show significant advantages under high speed loading, such as in impact and crash.

4.3.2 *Effect on n-Value and Total Elongation*

The effect of prestrains and strain rates on n-value and total elongation is shown in Fig. 18 to 23 for steels. The n-value decreases steadily with prestrain level at all strain rates. Total elongation decreases only slightly at equivalent prestrains less than approximately 4.5% and decreases dramatically at strains larger than 4.5%.

As shown in Fig. 8 to 10, the variation of uniform elongation with strain rate follows a very similar function as the n-value. Both n-value and uniform elongation decrease steadily with strain rate. However, the rate of decrease in n-value and uniform elongation varies with materials and is higher for low strength steels. The total elongation decreases slightly with strain rate for IF steels, AKDQ steels, and aluminum alloys and remains constant or even increases slightly with strain rates for HSLA steels in spite of the decrease in n-value and uniform elongation at higher strain rates. The fact that strain rate hardening increases with the strain rate, as observed above, partially compensates for the total elongation lost due to the decrease in strain hardening (uniform elongation). Thus, the total elongation is less dependent upon strain rate. The slight increase in total elongation with strain rate in some high strength steels is probably due to the lower dependence of uniform elongation (or n-value) on the strain rate than for AKDQ and IF steels. Thus, the gain of total elongation from strain rate hardening exceeds the loss due to the decrease in uniform elongation.

A statistical analysis also shows that uniform elongation measured in an ASTM standard tensile test depends primarily upon n-value only, while the postuniform elongation depends primarily upon the m-value only, as shown in Fig. 24 (the n-value and m-value here are measured in the strain range from 10% to the end of uniform elongation). However, the total elongation depends not only on the n-value but also on the m-value. These results agree with those observed by Ghosh (Ref 7, 23). The correlations of uniform elongation, postuniform elongation, and total elongation to n-value and m-value can be statistically determined and are given as follows:

Uniform elongation (%) = $104.9n - 1.2$ with $R^2 = 0.66$

Postuniform elongation (%) = $0.9 + 802.7m$ *with* $R^2 = 0.63$

Total elongation (%) = $935m + 109n - 3.5$ with $R^2 = 0.87$

where R^2 is the coefficient of correlation. Therefore, positive strain rate sensitivity (m-value) increases the capability of material in necking resistance (increasing the postuniform elongation) when compared to zero strain rate sensitivity.

Conclusions

- Strain rate sensitivity of steels and aluminum alloys decreases with an increase of uniaxial strain. The amount of decrease is very significant, particularly at low strain, for low strength steels and is relatively small for high strength steels and aluminum alloys.
- Strain hardening and strain rate hardening behaviors of steels vary with the strain. Different constitutive parameters should be used in different strain ranges.
- Yield and tensile strengths increase with prestrains and strain rates. The increase in tensile strength is dominated by strain rate hardening. The increase in yield strength from strain rate hardening is greater than that from strain hardening at low strain levels and is less at high strain levels.
- Both n-value and uniform elongation decrease with strain rate. Total elongation also decreases with strain rate for low strength steels and aluminum alloys but remains constant or even slightly increases with strain rate for high strength steels.

- The uniform elongation depends only on the n-value (strain hardening). The postuniform elongation depends only on the m-value (strain rate hardening). However, the total elongation depends upon both n-value and m-value.

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